

# ACTUARIAL EQUIVALENCE

## OR “THE PUSH”

1. Actuarial Tables and Shorthand Method of Determination\*
2. Article on Actuarial Equivalence by Donald W. Parkyn

\*Note: These actuarial tables developed by Donald W. Parkyn are somewhat out of date. Moreover, the selection of tables and interest rates is a function to be performed by an expert. Thus these tables should not be relied on for determining a cashout of benefits. For that purpose, one should contact an actuary or other professional skilled in valuing benefits in the context of divorce.

Copyright **Donald W. Parkyn and Barbara A. DiFranza**. May be reproduced for non-commercial purposes with attribution to the authors. Contact information may be found at [www.justpensions.com](http://www.justpensions.com) and [www.difranza.com](http://www.difranza.com).

## DiFranza's Explanation of Parkyn's Tables

Caution: This is a lawyer's explanation of an actuary's work product. It may make no sense to an engineer! You can use this to get an idea of the magnitude of a benefit value, but for a buyout, call the expert.

$X$  = Individual's Age

$E(X)$  = Life expectancy (actuaries argue against its use in evaluation work).

$L(X)$  = Relative number of people living. I.e., of 9836.220 people alive at age 20, there are 9225.182 alive at age 50. Of these, there are 6540.734 living at age 70.

$D(X)$  = Denominator = Choose based on current age.

$N... (X)$  = Numerator = Choose based on age that retirement starts. Numbers after the N indicate retirements with automatic cost of living. E.g.,  $N-2(X)$  = a retirement with a 2 percent annual COLA.

### Example of Actuarial Present Valuation:

\$250 a month will be paid starting at this 50 year old male's 55th birthday. Use 8 percent table.

\$250 x  $N-0(X)$  Retirement age 55

-----

$D(X)$  Current age 50

15037.277

\$250 x ----- = \$19,112.72

196.692

### Quick Method to Determine Actuarial Equivalence for Delayed Starting Date Starting today at age $X$ vs. starting later at age $X+n$

Pick the factor in the numerator column with the appropriate numerator for Participant's age at proposed starting date. E.g. age 62, no COLA 49,821.964

the factor in the same column opposite the age to which the benefit will be compared. E.g. age 65 is 37,067.503. Divide it by the above factor. This will give you the shrinkage factor. Result: .744 indicating a shrinkage of 25.6%.

The shrinkage factor can be ameliorated by any increase in benefit due to salary increases or increase in the benefit factor applicable to the Participant.

X	E(X)	L(X)	D(X)	N-0(X)	N-2(X)	N-3(X)	N-3.5(X)	N-4(X)	X
20	53.831	9836.220	3707.173	821871.528	1166038.223	1433381.076	1603310.174	1804505.849	20
21	52.898	9823.757	3526.167	778561.990	1100728.292	1349597.921	1507259.241	1693467.655	21
22	51.962	9811.782	3354.161	737366.021	1038770.371	1270306.322	1416497.957	1588731.984	22
23	51.022	9800.332	3190.712	698178.508	979995.704	1195273.527	1330745.733	1489955.253	23
24	50.080	9789.071	3035.281	660900.264	924244.658	1124279.126	1249737.594	1396813.189	24
25	49.136	9778.019	2887.481	625437.590	871366.235	1057114.688	1173222.387	1308998.626	25
26	48.190	9767.195	2746.938	591701.350	821216.605	993582.030	1100962.295	1226221.302	26
27	47.241	9756.617	2613.298	559606.755	773659.282	933493.207	1032731.371	1148205.851	27
28	46.292	9746.099	2486.173	529073.493	728564.626	876669.793	968315.905	1074691.897	28
29	45.341	9735.544	2365.219	500025.619	685809.990	822943.179	907513.499	1005433.329	29
30	44.391	9724.728	2250.087	472391.349	645279.117	772153.376	850132.588	940197.483	30
31	43.441	9713.632	2140.495	446102.650	606861.768	724148.833	795991.848	878764.242	31
32	42.492	9702.238	2036.176	421094.783	570452.975	678785.651	744919.085	820925.122	32
33	41.542	9690.518	1936.873	397306.143	535953.040	635927.032	696751.148	766482.702	33
34	40.595	9677.949	1842.248	374678.730	503267.695	595443.820	651333.667	715250.569	34
35	39.651	9664.420	1752.069	353157.915	472307.775	557213.734	608520.999	667052.682	35
36	38.711	9649.797	1666.113	332691.799	442988.644	521120.851	568174.868	621722.550	36
37	37.773	9633.943	1584.167	313231.090	415229.780	487055.048	530164.261	579102.432	37
38	36.839	9617.064	1506.088	294728.601	388954.366	454911.470	494364.594	539042.703	38
39	35.910	9598.330	1431.575	277139.881	364089.740	424591.015	460658.209	501402.286	39
40	34.985	9577.933	1360.508	260422.919	340567.049	395999.834	428933.756	466047.611	40
41	34.065	9555.646	1292.707	244537.534	318320.651	369048.534	399085.346	432852.188	41
42	33.151	9531.221	1228.002	229445.634	297288.136	343652.350	371012.770	401696.373	42
43	32.243	9504.362	1166.230	215111.123	277410.338	319731.020	344621.139	372466.966	43
44	31.340	9475.517	1107.325	201499.245	258630.583	297207.801	319820.030	345056.518	44
45	30.445	9443.205	1050.999	188577.466	240895.423	276010.425	296524.208	319363.793	45
46	29.559	9407.614	997.179	176315.310	224154.517	256070.699	274653.347	295293.412	46
47	28.680	9368.290	945.724	164683.619	208359.819	237323.690	254131.114	272754.947	47
48	27.812	9324.868	896.515	153654.785	193465.677	219707.871	234885.211	251662.963	48
49	26.952	9277.283	849.467	143202.415	179428.589	203164.673	216847.014	231936.415	49
50	26.101	9225.182	804.473	133301.272	166207.019	187638.343	199951.283	213498.561	50
51	25.260	9168.023	761.418	123927.457	153761.549	173076.110	184136.317	196276.845	51
52	24.431	9105.194	720.190	115058.425	142054.863	159427.961	169343.782	180202.825	52
53	23.613	9036.514	680.722	106672.689	131051.418	146646.468	155518.347	165211.698	53
54	22.806	8961.709	642.940	98749.612	120717.187	134686.321	142607.381	151241.974	54
55	22.009	8880.758	606.792	91269.293	111019.562	123504.282	130560.710	138235.225	55
56	21.223	8793.061	572.191	84212.693	101927.389	113059.228	119330.747	126136.136	56
57	20.450	8697.973	539.051	77561.814	93411.155	103312.207	108872.472	114892.467	57
58	19.689	8594.789	507.291	71299.641	85442.894	94226.394	99143.363	104454.969	58
59	18.941	8483.469	476.877	65409.837	77995.849	85766.654	90102.976	94776.937	59
60	18.206	8363.326	447.737	59876.724	71044.439	77899.539	81712.878	85814.077	60
61	17.485	8233.619	419.803	54685.453	64564.388	70593.388	73936.739	77524.625	61
62	16.778	8093.565	393.011	49821.964	58532.672	63818.204	66740.209	69869.146	62
63	16.088	7942.337	367.303	45272.934	52927.445	57545.599	60090.823	62810.459	63
64	15.415	7779.384	342.635	41025.640	47727.880	51748.581	53957.798	56313.384	64
65	14.756	7605.422	319.022	37067.503	42913.676	46401.083	48311.498	50344.198	65
66	14.119	7416.450	296.281	33387.054	38466.062	41478.899	43124.412	44871.615	66
67	13.500	7214.485	274.489	29973.331	34367.087	36958.981	38370.406	39865.967	67
68	12.897	7000.691	253.671	26814.784	30598.584	32818.392	34023.658	35298.166	68
69	12.308	6776.158	233.843	23899.617	27142.508	29034.620	30058.988	31140.024	69
70	11.733	6540.734	214.970	21216.178	23981.309	25585.962	26452.216	27364.578	70
71	11.172	6294.364	197.021	18753.205	21098.165	22451.723	23180.360	23946.275	71
72	10.627	6037.101	179.970	16499.779	18476.924	19612.145	20221.546	20860.882	72
73	10.099	5768.425	163.772	14445.422	16102.176	17048.476	17555.063	18085.522	73
74	9.588	5488.634	148.408	12580.020	13959.178	14742.859	15161.253	15598.539	74
75	9.095	5198.214	133.862	10893.668	12033.672	12678.161	13021.323	13379.317	75
76	8.622	4897.887	120.122	9376.629	10311.846	10837.908	11117.276	11408.186	76
77	8.170	4588.625	107.179	8019.294	8780.286	9206.231	9431.847	9666.365	77
78	7.735	4273.716	95.070	6811.857	7425.642	7767.525	7948.155	8135.583	78
79	7.318	3954.692	83.784	5744.378	6234.698	6506.514	6649.771	6798.164	79
80	6.921	3633.350	73.310	4807.049	5194.628	5408.500	5520.948	5637.234	80
81	6.393	3384.400	65.036	3981.111	4283.904	4450.254	4537.516	4627.611	81
82	6.021	3058.760	55.979	3259.552	3493.489	3621.451	3688.422	3757.460	82
83	5.663	2739.700	47.752	2641.278	2819.749	2916.956	2967.719	3019.968	83
84	5.320	2430.618	40.348	2116.381	2250.682	2323.528	2361.488	2400.500	84

X	E(X)	L(X)	D(X)	N-0(X)	N-2(X)	N-3(X)	N-3.5(X)	N-4(X)	X
20	59.358	9901.092	3731.622	843744.633	1220796.688	1523356.767	1719510.844	1955226.689	20
21	58.443	9886.705	3548.762	800153.756	1154137.417	1436680.035	1619258.821	1838121.252	21
22	57.527	9872.498	3374.917	758698.603	1090878.336	1354600.653	1524459.937	1727573.015	22
23	56.607	9858.538	3209.662	719273.756	1030849.442	1276882.493	1434827.758	1623228.293	23
24	55.685	9844.884	3052.587	681778.796	973888.959	1203300.935	1350090.778	1524752.204	24
25	54.760	9831.584	2903.299	646118.122	919843.151	1133642.850	1269990.879	1431826.730	25
26	53.831	9818.695	2761.422	612200.740	868565.533	1067705.515	1194283.586	1344151.262	26
27	52.898	9806.254	2626.594	579940.063	819917.095	1005296.664	1122736.560	1261440.516	27
28	51.962	9794.301	2498.468	549253.756	773765.470	946233.410	1055129.692	1183424.347	28
29	51.022	9782.871	2376.717	520063.519	729985.044	890342.545	991254.094	1109846.959	29
30	50.080	9771.630	2260.939	492295.472	688456.895	837459.835	930912.180	1040466.627	30
31	49.136	9760.598	2150.845	465879.815	649068.494	787429.937	873917.133	975054.829	31
32	48.190	9749.793	2046.156	440750.155	611712.734	740105.389	820091.545	913395.112	32
33	47.242	9739.234	1946.610	416843.336	576287.927	695346.011	769267.347	855282.488	33
34	46.292	9728.735	1851.916	394099.531	542697.580	653019.110	721285.113	800522.994	34
35	45.341	9718.199	1761.819	372462.172	510850.231	612998.857	675994.249	748933.222	35
36	44.391	9707.402	1676.059	351877.785	480659.365	575166.248	633252.029	700339.979	36
37	43.441	9696.326	1594.425	332295.697	452042.797	539408.352	592923.330	654579.232	37
38	42.492	9684.952	1516.719	313667.682	424922.424	505617.924	554879.938	611495.658	38
39	41.542	9673.253	1442.750	295947.854	399223.908	473693.131	519000.267	570942.293	39
40	40.595	9660.706	1372.265	279093.007	374877.029	443537.753	485169.466	532780.171	40
41	39.651	9647.201	1305.092	263062.449	351815.453	415060.727	453278.876	496878.252	41
42	38.711	9632.604	1241.065	247817.522	329976.035	388175.616	423225.577	463112.471	42
43	37.773	9616.778	1180.024	233321.509	309298.835	362800.485	394911.997	431365.306	43
44	36.839	9599.929	1121.864	219539.260	289726.658	338857.192	368245.361	401525.403	44
45	35.910	9581.229	1066.360	206437.666	271205.340	316271.914	343137.945	373487.557	45
46	34.985	9560.869	1013.423	193985.432	253683.620	294974.730	319506.816	347152.359	46
47	34.065	9538.621	962.919	182152.629	237112.594	274899.072	297273.161	322425.550	47
48	33.151	9514.240	914.722	170910.880	221445.760	255981.826	276362.282	299218.017	48
49	32.243	9487.429	868.709	160233.302	206639.073	238163.162	256703.525	277445.448	49
50	31.340	9458.634	824.831	150094.002	192650.288	221385.920	238229.499	257027.788	50
51	30.445	9426.380	782.875	140468.748	179439.625	205596.316	220876.773	237889.634	51
52	29.559	9390.852	742.785	131334.837	166969.564	190743.478	204585.477	219959.943	52
53	28.680	9351.599	704.457	122670.551	155204.306	176779.109	189298.745	203171.359	53
54	27.812	9308.254	667.802	114455.325	144109.866	163657.329	174962.757	187460.226	54
55	26.952	9260.754	632.756	106669.499	133653.844	151334.521	161526.339	172766.197	55
56	26.101	9208.746	599.241	99294.275	123805.282	139769.186	148940.949	159032.107	56
57	25.260	9151.688	567.169	92311.850	114534.823	128921.965	137160.593	146203.888	57
58	24.431	9088.972	536.460	85705.431	105814.672	118755.666	126141.837	134230.566	58
59	23.613	9020.414	507.060	79459.011	97618.357	109234.901	115843.464	123063.888	59
60	22.806	8945.743	478.917	73557.221	89920.543	100325.951	106226.263	112658.032	60
61	22.009	8864.936	451.991	67985.234	82696.916	91996.616	97252.875	102969.498	61
62	21.223	8778.394	426.217	62728.871	75924.281	84216.236	88887.825	93957.044	62
63	20.450	8682.476	401.531	57774.723	69580.656	76955.821	81097.596	85581.790	63
64	19.689	8579.475	377.874	53110.117	63645.216	70187.920	73850.522	77807.052	64
65	18.941	8468.354	355.219	48722.883	58098.014	63886.381	67116.463	70598.016	65
66	18.206	8348.425	333.513	44601.344	52920.006	58026.280	60866.789	63921.710	66
67	17.485	8218.950	312.705	40734.438	48093.107	52584.025	55074.449	57747.011	67
68	16.778	8079.145	292.749	37111.693	43600.165	47537.287	49713.857	52044.551	68
69	16.088	7928.186	273.599	33723.182	39424.912	42864.911	44760.825	46786.635	69
70	15.415	7765.524	255.224	30559.432	35551.827	38546.794	40192.415	41947.054	70
71	14.756	7591.871	237.635	27611.071	31965.795	34563.518	35986.565	37500.692	71
72	14.119	7403.236	220.696	24869.555	28652.830	30897.051	32122.780	33424.240	72
73	13.500	7201.631	204.463	22326.720	25599.559	27530.228	28581.585	29695.603	73
74	12.897	6988.218	188.956	19973.962	22792.455	24445.961	25343.752	26293.113	74
75	12.308	6764.085	174.186	17802.494	20218.073	21627.483	22390.523	23195.771	75
76	11.733	6529.080	160.128	15803.638	17863.341	19058.626	19703.890	20383.491	76
77	11.172	6283.149	146.759	13969.003	15715.727	16723.974	17266.730	17837.247	77
78	10.627	6026.344	134.057	12290.457	13763.202	14608.813	15062.749	15538.980	78
79	10.099	5758.148	121.992	10760.195	11994.286	12699.171	13076.524	13471.652	79
80	9.588	5478.855	110.547	9370.683	10397.996	10981.750	11293.408	11619.134	80
81	9.095	5188.952	99.712	8114.543	8963.713	9443.785	9699.404	9966.066	81
82	8.622	4889.160	89.478	6984.521	7681.149	8073.006	8281.105	8497.799	82
83	8.170	4580.449	79.836	5973.461	6540.312	6857.593	7025.652	7200.340	83
84	7.735	4266.102	70.816	5074.058	5531.256	5785.921	5920.471	6060.083	84

## Actuarial Equivalence

Two benefit streams are actuarial equivalent if they have the same actuarial present value. Actuarial present value is based on assumptions. The assumptions used by many retirement plans to determine “actuarial equivalence” are not reasonable. In theory it should be possible for a plan to with a large number of male employees all aged 50 to be split into two groups and provide \$X per month for life to group 1 starting at age 50 and \$Y per month to group 2 starting at age 50 and the cost of providing benefits to group 1 would be the same as providing such benefits to group 2.

In the gaming world, the term “push” is equivalent to the actuarial term “actuarial equivalence.” Michael Shackelford, known as the Wizaardofodds, was an actuary with the Social Security Administration for several years before coming to Las Vegas where he applies actuarial methods to estimate the house “edge” in various casino games. The math involved is the same as that used in retirement calculations. See [www.wizardofodds.com](http://www.wizardofodds.com).

Often a retirement plan will provide an early retirement subsidy to encourage plan members to retire early – they get rid of the older, higher paid employees and replace them with younger lower paid employees and in so doing reduce retirement plan costs. Other plans such as the CalPERS plan for State miscellaneous members over-reduces the reduction for early retirement while at the same time providing a super generous early retirement for the CHP plan which allows retirement at 3% per year for retirement at age 50 but no additional increase in the benefit factor for retirement at later ages.

In the casino environment terms like “push,” “edge,” and “comps” are used to describe events very much like those found in retirement plans. “Push” is another way to describe “actuarial equivalence.” “Edge” is akin to a negative subsidy. “Comp” is akin to an early retirement subsidy.

### Actuarial Tables for Calculation of Actuarial Values

The following two tables, known as actuarial commutation functions, can be used to calculate the actuarial present value of pension benefits. One table is for males and the other for females.

**The rule** is: numerator value at retirement age over denominator at attained age times monthly pension amount equals the actuarial present value. The commutation functions

are based on the UP-84 mortality table, set forward one year for males and setback five years for females, and an interest rate of 5.00%.

**To use:**

A value for the attained age will be used from the D(X) column as the **denominator** in your calculation.

A value, based on the retirement age, from one of the columns labeled N-0(X), N-2(X), N-3(X), N-3.5(X), and N-4(X) will be used for the **numerator** in your calculation.

The column selected for the numerator will depend on the post-retirement cost-of-living provision of the pension. If no post-retirement cost-of-living select a value from the column headed N-0(X), for post-retirement cost-of-living of 2.00% select a value from the column headed N2(x), etc.

For example, the actuarial present value of a pension without post-retirement cost-of-living of \$1 per month for a male age 50 who will retire at age 60 has a numerator of 59876.724 (entry for age 60 from the column labeled N-0(X) and a denominator of 804.473 (entry for age 50 from the column labeled D(X)).  $59876.724/804.473 = \mathbf{\$74.43}$ , the lump sum necessary now (the actuarial present value factor) to provide \$1 per month starting at age 60 to a male now age 50.

Some retirement plans, mostly found in public sector plans, provide post retirement cost of living increases (COLA's). If a post retirement COLA of 2% is involved a numerator reflecting this is used. If a 2% COLA is involved the example give above the numerator of 59876.724 is replaced by the value for age 60 from the column labeled N-2(X) which is 71044.439 .  $71044.439/804.473 = \mathbf{\$88.31}$ , the lump sum necessary now to provide \$1 per month starting at age 60 to a male now age 50..

The charts/table attached can be used to determine what is a reasonable reduction for early retirement.

For example, a lump sum needed to provide\$1 per month starting at age 50 to a male now age 50 with a 2% COLA is  $166207.019/804.473 = 206.60$ .

To establish an actuarial equivalence we need to factor-down the \$1 per month starting at age 60 to a lesser amount starting at age 50. This is done by  $88.31/206.60 = 43$  cents. However, Nevada PERS reduces the benefit by 4% per year which is 40% for the 10 year early retirement allowing the member to receive 60 cents per month.

**A shortcut for this calculation** is to take the ratio of the numerator values. In this case,  $71044.439/166207.619 = 43\%$  is what the member gets. The reduction is 57%.

### **Nevada PERS**

Nevada PERS has an atypical scheme for determining retirement. When a non-safety member has five years of service he/she has a “normal retirement age” of 65. However, when he/she has 10 years of service his/her “normal retirement age” is reduced to age 60. A member can retire before his/her “normal retirement age” with a 4% per year reduction for each year he/she is under his/her “normal retirement age.” However, when he/she has 30 years of service he/she has reached his/her “normal retirement age.” If you have a case involving a member who was hired at 18 and is now age 47 he/she has a “normal retirement age” of 60 but can retire at age 47--13 years early--but his/her benefit will be reduced by 52%. However, if he/she works just one more year he/she can retire without reduction. Practice tip: watch out for this or face a possible malpractice action.

\* \* \*